Weekly Assignment 4 (Selected) Solutions (Due Tuesday 7/20 11:59PM)

Overview: This assignment is graded out of **96 points**. There are **7 problems**, each of which is worth **16 points** for a total of **112 points** possible. **Therefore, a score above 96 points earns you extra credit.** Your score for each question will depend on the graders determination of your proficiency in each of the following categories: Conceptual Understanding, Strategies & Reasoning, Computation & Execution, and Communication. You can earn up to 4 points for each category. The grader determines your score for each category using the Weekly Assignment Rubric.

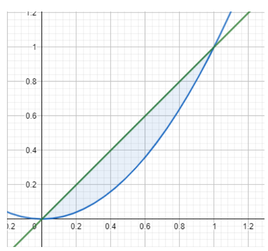
Guidelines: You are required to adhere to the weekly assignment guidelines and the assignment submission guidelines in the syllabus. If you fail to follow the guidelines, you risk receiving no credit for you work. Turn in your assignment via gradescope.

Directions: Complete the following exercises from the Active Calculus textbook. You can click the links below to go directly to the exercise.

1. Exercise 10.7.21.
2. Exercise 10.8.13.
3. Exercise 11.1.10.
4. Exercise 11.2.12.
5. Exercise 11.3.11.

Solution. I am not going to show any of the computations, because it is too time consuming to type. In your solution, you should have provided the details.

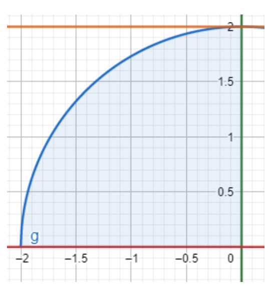
(a) Here's the region



Interpreting the region as a Type II region, the integral can be written

$$\int_0^1 \int_{x^2}^{x} xy\,dy\,dx = \int_0^1 \int_y^{\sqrt{y}} xy\,dx\,dy = 1/24.$$

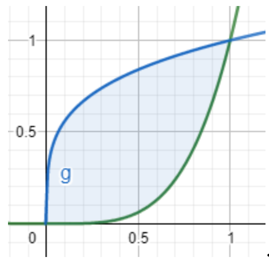
(b) Here's the region:



Interpreting this as a Type I region, the integral can be written

$$\int_0^2 \int_{-\sqrt{4-y^2}}^0 xy\,dx\,dy = \int_{-2}^0 \int_0^{\sqrt{4-x^2}} xy, dy\,dx = -2.$$

(c) Here's the region



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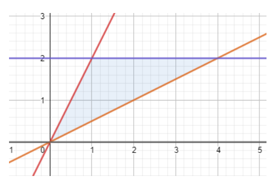
Interpreting this as a Type II region, the integral becomes

$$\int_0^1 \int_{x^4}^{x^{1/4}} x + y\,dy\,dx = \int_0^1 \int_{y^4}^{y^{1/4}} x + y\,dx\,dy = 5/9.$$

Notice how the integrals are exactly the same, but with x, y switched. The reason is because both the region and the graph of the function are symmetric about the line $y = x$.

(d) Here's the region



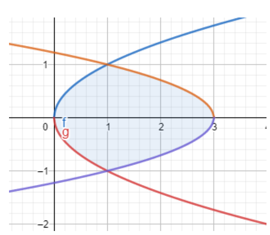
The integral as written uses s description of the region as Type II. However, the region is NOT Type I. It is, however, a union of Type I regions and therefore the double integral can be computed using two integrals. We have

$$\int_0^2 \int_{y/2}^{2y} x + y\,dx\,dy = \int_0^1 \int_{x/2}^{2x} x + y\,dy\,dx + \int_1^4 \int_{x/2}^{2} x + y\,dy\,dx = 9.$$

□

6. Exercise 11.3.12.

Solution. **(a)** Here's the region



(b) We have

$$\iint_D T(x,y)\,dA = \int_{-1}^{1}\int_{y^2}^{3-2y^2} 100 - 4x^2 - y^2\,dx\,dy.$$

(c) This region can be viewed as a union of two Type I regions. We have

$$\iint_D T(x,y)\,dA = \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} 100 - 4x^2 - y^2\,dy\,dx + \int_1^3 \int_{-\sqrt{(3-x)/2}}^{\sqrt{(3-x)/2}} 100 - 4x^2 - y^2\,dy\,dx.$$

(d) After a long computation, I found that

$$\iint_D T(x,y)\,dA = \int_{-1}^{1}\int_{y^2}^{3-2y^2} 100 - 4x^2 - y^2\,dx\,dy = 2516/7.$$

(e) Find the area of the region using Calc I. We have

$$A = \int_{-1}^{1} (3-y^2) - y^2\,dy = 4.$$

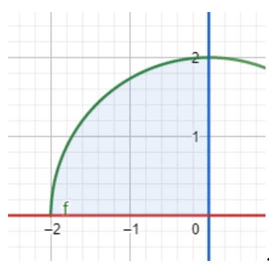
Thus, the average temperature on the region D is given by

$$T_{\text{AVG}(D)} = \frac{1}{A}\iint_D T(x,y)\,dA = 2516/28 \text{ degrees Celsius.}$$

□

7. Exercise 11.3.13.

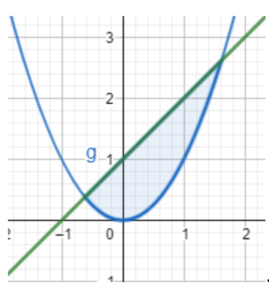
Solution. **(a)** Here's the region



The volume of the solid is given by

$$\int_{-2}^{0}\int_{0}^{\sqrt{4-x^2}} 16 - x^2 - y^2\,dy\,dx.$$

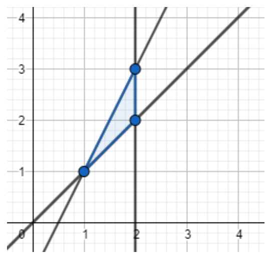
(b) Here's the region



The volume of the solid is given by

$$\int_{\frac{1-\sqrt{5}}{2}}^{\frac{1+\sqrt{5}}{2}} \int_{x^2}^{x+1} 10 - x - 2y \, dy \, dx.$$

(c) The region looks like

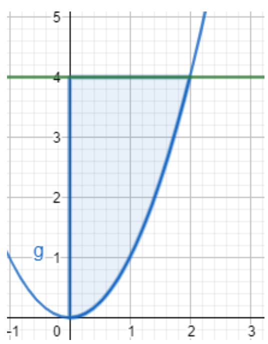


The volume of the solid is given by

$$\int_1^2 \int_x^{2x-1} e^{-xy} dy \, dx.$$

I found this by computing the equation of the lines determined by the three points.

(d) Here's the region



The volume of the solid is given by

$$\int_0^2 \int_{x^2}^4 \sqrt{1 + x^2 + y^2} \, dy \, dx.$$

□